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On transverse spin sum rules

A. Harindranath [a], Rajen Kundu [b], Asmita Mukherjee [c,*]

[a] Theory Division, Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India
[b] Department of Physics, RKMVC College, Rahara, Kolkata, 700118, West Bengal, India
[c] Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

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ABSTRACT

In this work we show that (i) both the form factors A_i and $\bar{C}_i$ contribute to the matrix element of the energy–momentum tensor T_i^{+-} in a transversely polarized state, (ii) there is no relative suppression factor between these two contributions and (iii) the contribution to the matrix element of the Pauli–Lubanski operator $W_i^{\perp}$ from that of T_i^{++} contains only the form factor B_i and not the form factor A_i. These results support our criticism and the conclusions as stated in Ref. [13]. Comparing and contrasting the spin sum rules in two different approaches, one advocated by us and the one proposed by Jaffe and Manohar, we point out that the physical content of the sum rules is very transparent in our approach, whereas, in the second approach details of the dynamics remain hidden and the separation into orbital and intrinsic spin parts is not visible.

Introduction

At present, understanding the helicity and the transverse spin structure of the proton in the context of Deep Inelastic Scattering (DIS) is of great interest. Intense experimental and theoretical research activities have been going on in this field for more than a decade. It is well-known that since DIS is a light cone dominated process, the most appropriate theoretical tool to study it is provided by Light Front Quantization (for a review, see Ref. [1]). In order to understand the spin structure of the proton which is a composite object and investigate any sum rule associated with it, one should start from the intrinsic spin operators $\mathcal{J}^i$, $i = 1, 2, 3$, which can be constructed from the Pauli–Lubanski operator. Among the Poincare group generators, the intrinsic spin operators on the light front commute with the generators of translations and boosts (which are kinematical as well in the light front dynamics) in the longitudinal and transverse directions. As a result, the light front intrinsic spin operators are boost and translation invariant and, further, they obey the angular momentum algebra [2–4]. On the other hand, instant form intrinsic spin operators do not commute with boost operators which are dynamical [5]. Any angular momentum sum rule, solely based on the matrix elements of rotation operators that are part of Poincare generators, will have frame dependence. The same is also true, in general, if one starts with the Pauli–Lubanski operators as we discuss below. As already stated, the solution to this problem is to start from the intrinsic spin operators $\mathcal{J}^i$.

The helicity operator $\mathcal{J}^3$ (whose explicit construction and a perturbative analysis in light front QCD is carried out in Ref. [6] in the total transverse momentum zero frame) is kinematical (interaction free). On the other hand, it is well known that the transverse rotation operators and hence the transverse spin operators in light front theory are dynamical (interaction dependent). Construction and analysis of $\mathcal{J}^i$ $(i = 1, 2)$ in light front QCD is carried out in Refs. [7,8].

Recently, the matrix element of the transverse component of the Pauli–Lubanski operator has been formally analyzed in Refs. [9] and [10] (hereafter referred to as Ji et al.) following the approach of Ref. [11] and using the parameterizations of the off-forward matrix elements of the energy–momentum tensor. These authors are partly inspired by Ref. [12] in which a relation between the expectation value of equal time transverse rotation generator J_q^i and the form factors $A_q(0)$ and $B_q(0)$ is obtained using delocalized wave packet states that are transversely polarized in the rest frame of the nucleon.

We have pointed out in Ref. [13] that many of the statements in Ji et al. appear unsupported by explicit calculations. In this work we present explicit calculations supporting our statements in Ref. [13]. We also compare and contrast our approach [6–8] with the approach presented in [11] to derive sum rules.

* Corresponding author.
E-mail addresses: a.harindranath@saha.ac.in (A. Harindranath), rajenkundu@yahoo.com (R. Kundu), asmita@phy.iitb.ac.in (A. Mukherjee).

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General outline of the calculation

The starting point in Ji et al. is the Pauli–Lubanski operator which is defined in terms of energy–momentum tensor in a very standard way as follows.

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} M_{\nu\alpha} P_\beta,$$
$$M^{\mu\nu} = \frac{1}{2}\int dx^- d^2x^\perp \left[x^\mu T^{+\nu} - x^\nu T^{+\mu}\right]. \tag{1}$$

Whereas, starting point in Refs. [7,8] is the intrinsic spin operators which, for a massive particle like nucleon, are related to Pauli–Lubanski operators.

$$M\mathscr{J}^i = W^i - P^i \mathscr{J}^3$$
$$= \epsilon^{ij}\left(\frac{1}{2}F^j P^+ + K^3 P^j - \frac{1}{2}E^j P^-\right) - P^i \mathscr{J}^3,$$
$$\mathscr{J}^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}\left(E^1 P^2 - E^2 P^1\right). \tag{2}$$

In Eqs. (2), $F^i = M^{-i}$ are the light front transverse rotation operators and are interaction dependent or dynamical; while $E^i = M^{+i}$ are light front transverse boost operators and are interaction independent or kinematical. Longitudinal boost $K^3 = M^{+-}$ and helicity $J^3 = M^{12}$ are also kinematical. Note that the light front transverse rotation and the boost operators were mis-identified in Ji et al. This was already pointed out in Ref. [14]. Moreover, Ji et al. did not consider longitudinal boost operator $K^3 = M^{+-}$ for working explicitly in $P^\perp = 0$ frame and only for such a choice of frame, both the starting points appear to be the same. In the following, we kept this term to show an example in the course of our explicit calculations that, in general, for a frame with non-zero $P^\perp$ both are not the same. We also assume that the various Poincare generators can be separated to quark and gluon parts.

Next, to compare with the results of Ji et al., we need to calculate the transverse component of the Pauli–Lubanski operator corresponding to species i formally defined as

$$W_i^1 = \frac{1}{2}F_i^2 P^+ + K_i^3 P^2 - \frac{1}{2}E_i^2 P^- \tag{3}$$

and its matrix element in a transversely polarized state

$$\frac{\langle PS^{(1)}|W_i^1|PS^{(1)}\rangle}{(2\pi)^3 2P^+\delta^3(0)} \tag{4}$$

where i denotes either the quark or gluon part. Note that, in the rest of the Letter, we always deal with only one component, namely, W_i^1, while calculation with W_i^2 is trivially the same and unnecessary for our purpose.

The transverse rotation operator is

$$F_i^2 = \frac{1}{2}M_i^{-2} = \frac{1}{4}\int dx^- d^2x^\perp \left[x^- T_i^{+2} - x^2 T_i^{+-}\right]. \tag{5}$$

We note that,

$$K_i^3 = \frac{1}{2}M_i^{+-} = \frac{1}{4}\int dx^- d^2x^\perp \left[x^+ T_i^{+-} - x^- T_i^{++}\right]$$
$$= \frac{1}{2}x^+ P^- + \tilde{K}_i^3,$$
$$E_i^2 = M_i^{+2} = \frac{1}{2}\int dx^- d^2x^\perp \left[x^+ T_i^{+2} - x^2 T_i^{++}\right]$$
$$= x^+ P^2 + \tilde{E}_i^2. \tag{6}$$

In writing the last equalities in both the above expressions, we note that light front time x^+ can be taken out of the integral in the first terms and simplified. Putting them back in Eq. (3), we see that only the second terms in these expressions contribute to W_i^1. Thus we find that

$$W_i^1 = \frac{1}{2}F_i^2 P^+ + \tilde{K}^3 P^2 - \frac{1}{2}\tilde{E}_i^2 P^- \tag{7}$$

with no explicit x^+ dependence. Lastly, the light front helicity operator is given by

$$J_i^3 = M_i^{12} = \frac{1}{2}\int dx^- d^2x^\perp \left[x^1 T_i^{+2} - x^2 T_i^{+1}\right]. \tag{8}$$

According to the procedure prescribed in Ref. [11], rest of the calculation relies on defining the Fourier transform of the off-forward matrix elements of relevant component of energy–momentum tensor and then consider the forward limit. Since W_i^1 is independent of x^+ explicitly, we consider three dimensional Fourier transform of the off-forward matrix element. In general, we define

$$\left\langle P'S^{(1)}\middle|\hat{\mathscr{O}}^\alpha(k_-,k_i)\middle|PS^{(1)}\right\rangle$$
$$= \frac{1}{2}\int dx^- d^2x^\perp e^{i(k_-x^- + k_i x^i)} x^\alpha \left\langle P'S^{(1)}\middle|\mathscr{O}(x)\middle|PS^{(1)}\right\rangle \tag{9}$$

where $\alpha = -, 1, 2$. Using translational invariance, we find

$$\left\langle P'S^{(1)}\middle|\hat{\mathscr{O}}^\alpha(k)\middle|PS^{(1)}\right\rangle$$
$$= -i(2\pi)^3\frac{\partial}{\partial k_\alpha}\left[\delta^3\left(k + P' - P\right)\left\langle P'S^{(1)}\middle|\mathscr{O}(0)\middle|PS^{(1)}\right\rangle\right]$$
$$= -i(2\pi)^3\delta^3\left(k + P' - P\right)\frac{\partial}{\partial k_\alpha}\left\langle P'S^{(1)}\middle|\mathscr{O}(0)\middle|PS^{(1)}\right\rangle \tag{10}$$

ignoring the term containing the derivative on the delta function [11].

Thus, with $\Delta = P' - P$, we find

$$\left\langle PS^{(1)}\middle|{F_i}^2\middle|PS^{(1)}\right\rangle = i(2\pi)^3\delta^3(0)\left[\frac{\partial}{\partial\Delta_-}\left\langle P'S^{(1)}\middle|T_i^{+2}(0)\middle|PS^{(1)}\right\rangle\right.$$
$$\left.- \frac{\partial}{\partial\Delta_2}\left\langle P'S^{(1)}\middle|T_i^{+-}(0)\middle|PS^{(1)}\right\rangle\right]_{\Delta=0}, \tag{11}$$

$$\left\langle PS^{(1)}\middle|\tilde{K}_i^3\middle|PS^{(1)}\right\rangle$$
$$= -\frac{i}{2}(2\pi)^3\delta^3(0)\left[\frac{\partial}{\partial\Delta_-}\left\langle P'S^{(1)}\middle|T_i^{++}(0)\middle|PS^{(1)}\right\rangle\right]_{\Delta=0} \tag{12}$$

and

$$\left\langle PS^{(1)}\middle|\tilde{E}_i^2\middle|PS^{(1)}\right\rangle$$
$$= -i(2\pi)^3\delta^3(0)\left[\frac{\partial}{\partial\Delta_2}\left\langle P'S^{(1)}\middle|T_i^{++}(0)\middle|PS^{(1)}\right\rangle\right]_{\Delta=0}. \tag{13}$$

Matrix elements of the energy–momentum tensor

We start from the following parameterization as used in Ji et al.,

$$\left\langle P',S'\middle|T_i^{\mu\nu}(0)\middle|PS\right\rangle$$
$$= \bar{U}\left(P',S'\right)\left[A_i\left(\Delta^2\right)\frac{1}{2}\left(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu\right)\right.$$
$$+ B_i\left(\Delta^2\right)\frac{1}{2M_N}\frac{1}{2}\left(\bar{P}^\mu i\sigma^{\nu\alpha}\Delta_\alpha + \bar{P}^\nu i\sigma^{\mu\alpha}\Delta_\alpha\right)$$
$$\left.+ C_i\left(\Delta^2\right)\frac{1}{M_N}\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right) + \bar{C}_i\left(\Delta^2\right)M_N g^{\mu\nu}\right]U(P,S). \tag{14}$$

Here $\bar{P} = \frac{1}{2}(P + P')$.

Note that, in the above parameterization, effects of QCD-interactions are buried in the form factors while the associated Lorentz structures are given in terms of asymptotic spin-half nucleonic states. We can either calculate the matrix elements in Eq. (14) directly (which we denote by case I) or use the Gordon identity

$$\overline{U}(P',S')\frac{i}{2M_N}\sigma^{\mu\nu}\Delta_\nu U(P,S) = \overline{U}(P',S')\gamma^\mu U(P,S) - \overline{U}(P',S')\frac{(P+P')^\mu}{2M_N}U(P,S) \tag{15}$$

to eliminate either the "σ" terms (case II) or the "γ" terms (case III which is used in Ji et al.) from Eq. (14).

Eliminating the "σ" terms (case II) we have

$$\begin{aligned}\langle P',S'|T_i^{\mu\nu}(0)|PS\rangle &= \overline{U}(P',S')\bigg[-B_i(\Delta^2)\frac{\overline{P}^\mu\overline{P}^\nu}{M_N} + \big(A_i(\Delta^2)+B_i(\Delta^2)\big)\frac{1}{2}(\gamma^\mu\overline{P}^\nu+\gamma^\nu\overline{P}^\mu)\\ &\quad + C_i(\Delta^2)\frac{1}{M_N}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2) + \overline{C}_i(\Delta^2)M_N g^{\mu\nu}\bigg]U(P,S).\end{aligned} \tag{16}$$

On the other hand, eliminating the "γ" terms (case III) we have

$$\begin{aligned}\langle P',S'|T_i^{\mu\nu}(0)|PS\rangle &= \overline{U}(P',S')\bigg[A_i(\Delta^2)\frac{\overline{P}^\mu\overline{P}^\nu}{M_N} + \big(A_i(\Delta^2)+B_i(\Delta^2)\big)\frac{1}{2M_N}\frac{1}{2}(\overline{P}^\mu i\sigma^{\nu\alpha}\Delta_\alpha+\overline{P}^\nu i\sigma^{\mu\alpha}\Delta_\alpha)\\ &\quad + C_i(\Delta^2)\frac{1}{M_N}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2) + \overline{C}_i(\Delta^2)M_N g^{\mu\nu}\bigg]U(P,S).\end{aligned} \tag{17}$$

All the three cases should yield the same results since Eq. (15) is simply an identity which is valid for on-shell spin-half states.

We present explicit calculation in case II. Further details of the calculation are given in the Appendix that are used to obtain the results given below.

In the following we keep only the terms which are linear in Δ, which are relevant for the computation of the matrix elements of the transverse spin. Then, the matrix elements of $T^{\mu\nu}(0)$ in the transversely polarized state (to be specific, taken to be polarized along +ve x direction) are (in the frame $\overline{P}^\perp = 0$)

$$\langle P',S^{(1)}|T_i^{++}(0)|PS^{(1)}\rangle = -B_i(\Delta^2)\frac{\overline{P}^+\overline{P}^+}{M_N}(-i\Delta^{(2)}), \tag{18}$$

$$\langle P',S^{(1)}|T_i^{+1}(0)|PS^{(1)}\rangle = 0, \tag{19}$$

$$\langle P',S^{(1)}|T_i^{+2}(0)|PS^{(1)}\rangle = \frac{1}{2}\big(A_i(\Delta^2)+B_i(\Delta^2)\big)(-i)M_N\Delta^+, \tag{20}$$

$$\langle P',S^{(1)}|T_i^{+-}(0)|PS^{(1)}\rangle = -A_i(\Delta^2)iM_N\Delta^{(2)} + \overline{C}_i(\Delta^2)M_N g^{+-}\big((-)i\Delta^{(2)}\big). \tag{21}$$

From Eq. (18) we re-confirm that $A_i(\Delta^2)$ does not appear in the matrix element of T^{++} in a transversely polarized state [17].

Cases I and III also yield the same results as Eqs. (18)–(21) in their dependence on A_i, B_i and $\overline{C}_i$. Note, however, that in these cases, Δ^- appears which we need to evaluate and replace. Since P and P' are on mass shell, Δ^- is related to Δ^+ and $\Delta^\perp$ by

$$\Delta^- = -\frac{\Delta^+}{(\overline{P}^+)^2-(1/4)(\Delta^+)^2}\left(M^2+\frac{1}{4}(\Delta^\perp)^2\right) \Rightarrow \quad -\Delta^+\frac{M^2}{(P^+)^2}. \tag{22}$$

We ignore the $(\Delta^\perp)^2$ term since we are interested only in the terms linear in Δ. We also need to use Eq. (31).

We summarize the results obtained so far, as follows.

1) In T^{++} matrix element, coefficient of A_i form factor vanishes and hence it depends only on B_i form factor.

2) T^{+2} matrix element depends only on Δ^+ explicitly and

3) T^{+-} matrix element depends only on A_i and $\overline{C}_i$ form factors.

Matrix elements of the Pauli–Lubanski operator W_i^1

Substituting the results for individual matrix elements, in Eq. (4) we get

$$\begin{aligned}\frac{\langle PS^{(1)}|W_i^1|PS^{(1)}\rangle}{\langle PS^{(1)}|PS^{(1)}\rangle} &= \frac{1}{2P^+}\left[\frac{P^+}{2}\big(2A_i(0)+B_i(0)+2\overline{C}_i(0)\big)M_N + \frac{P^-}{2}\frac{(P^+)^2}{M_N}B_i(0)\right]\\ &= \frac{1}{2}M_N\big(A_i(0)+B_i(0)+\overline{C}_i(0)\big).\end{aligned} \tag{23}$$

Thus the matrix elements of T_i^{+2} and T_i^{+-} make comparable contributions to the matrix element of W_i^1 in a transversely polarized state. The matrix element of T^{++} does not contribute to the matrix element of total W^1.

The non-vanishing contribution of $\overline{C}_i$ to the matrix elements of W_i^1 has been noted previously by Hatta et al. [18] and Leader [19]. However, Ref. [18] used light front operators but equal time spinors and Ref. [19] used equal-time operators and equal-time spinors. As a result, Refs. [18] and [19] obtained different longitudinal momentum dependence for $(A_i + B_i)$ and $\overline{C}_i$, whereas we have obtained the same dependence for the form factors. In the infinite momentum limit, the result of Ref. [18] agrees with us.

Comment on the frame dependence of W_i^1 matrix elements

From the definitions of the intrinsic spin operators, it is clear that in $P^\perp = 0$ frame irrespective of the polarization S,

$$M_N\langle PS|\mathcal{J}_i^1|PS\rangle = \langle PS|W_i^1|PS\rangle, \tag{24}$$

$$\langle PS|\mathcal{J}_i^3|PS\rangle = \langle PS|J_i^3|PS\rangle. \tag{25}$$

Note that in Appendix D of Ref. [8], the calculation of the matrix element of the intrinsic transverse spin operator in a transversely polarized dressed quark state in an arbitrary reference frame is presented and the frame independence is explicitly demonstrated. The claim of the frame independence of results by the authors of Ji et al., by calculating the RHS of the above equations in the $P^\perp = 0$ frame only, is invalid as we demonstrate in the following. Extending the calculation presented in the last section for a frame with non-zero $P^\perp$ (i.e., not putting $P^\perp = 0$ from the very beginning) one could easily show that even though the LHS of the above equations are frame independent, while the RHS are not necessarily frame independent.

An explicit calculation shows that, for a transversely polarized nucleon,

$$\frac{\langle PS^{(1)}|\mathcal{J}_i^1|PS^{(1)}\rangle}{\langle PS^{(1)}|PS^{(1)}\rangle} = \frac{1}{2}\big(A_i(0)+B_i(0)+\overline{C}_i(0)\big) \tag{26}$$

and for a longitudinally polarized nucleon

$$\frac{\langle PS|\mathcal{J}_i^3|PS\rangle}{\langle PS|PS\rangle} = \frac{1}{2}\big(A_i(0) + B_i(0)\big), \tag{27}$$

which are frame independent. We also get

$$\frac{\langle PS|\mathcal{J}_i^1|PS\rangle}{\langle PS|PS\rangle} = 0,$$
$$\frac{\langle PS^{(1)}|\mathcal{J}_i^3|PS^{(1)}\rangle}{\langle PS^{(1)}|PS^{(1)}\rangle} = 0. \tag{28}$$

Results in Eqs. (28) are frame independent and correctly represent the fact that the expectation value of the helicity in a transversely polarized nucleon must be zero and the expectation value of intrinsic transverse spin in a longitudinally polarized nucleon must be zero. On the other hand, the RHS of the corresponding equations as obtained from Eq. (24) and Eq. (25) are frame dependent and do not reflect the correct results:

$$\frac{\langle PS|W_i^1|PS\rangle}{\langle PS|PS\rangle} = \frac{1}{2}\big(A_i(0) + B_i(0)\big)\overline{P}^1,$$
$$\frac{\langle PS^{(1)}|J_i^3|PS^{(1)}\rangle}{\langle PS^{(1)}|PS^{(1)}\rangle} = -B_i\frac{\overline{P}^1}{2M_N}. \tag{29}$$

We get vanishing the RHS only in the frame $P^\perp = 0$.

Comparison of two approaches

In this section we compare and contrast two approaches to the spin sum rules, namely the approach based on light front spin operators advocated by us and the one proposed by Jaffe and Manohar based on matrix elements of the energy–momentum tensor. In Refs. [7] and [8] we have presented a transverse spin sum rule for the nucleon in QCD using light front dynamics and intrinsic (boost invariant) transverse spin operators. The analysis relies on the explicit structure of Poincare generators and hence depends on the details of QCD dynamics since transverse spin operators in the light front theory are interaction dependent. In the gauge $A^+ = 0$ and using the equations of constraints, we were able to separate the operator into terms with and without explicit coordinate dependence. The latter could be further separated into quark ($\mathcal{J}_{II}^i$) and gluon parts ($\mathcal{J}_{III}^i$). What is the phenomenological relevance of this separation? We have demonstrated [7,8] that the nucleon matrix element of $\mathcal{J}_{II}^i$ is directly related to the integral of the well-known transverse polarized structure function g_T and the nucleon matrix element of $\mathcal{J}_{III}^i$ is directly related to the integral of the gluon distribution function that appears in transverse polarized hard scattering [20]. In the case of helicity, we have demonstrated similar connections [6], namely nucleon matrix element of ($\mathcal{J}_{q(i)}^3$) is directly related to the polarized structure function g_1 and the nucleon matrix element of ($\mathcal{J}_{g(i)}^3$) is directly related to the gluon distribution relevant to nucleon helicity [21]. Thus the physical content of our sum rules is very transparent.

On the other hand, the sum rule following Ref. [11] contains form factors that parameterize the off-forward matrix elements of the energy–momentum tensor. The details of the dynamics remain hidden in this formalism. The separation into orbital and intrinsic spin parts is not visible and relation of the sum rules to the quark and gluon helicity and transverse spin distribution functions that appear in various deep inelastic processes remain obscure.

Conclusions

We have found that (i) both the form factors A_i and $\bar{C}_i$ contribute to the matrix element of the energy–momentum tensor T_i^{+-} in a transversely polarized state, (ii) there is no relative suppression factor between these two contributions and (iii) the contribution to the matrix elements of Pauli–Lubanski operator $W_i^\perp$ from that of T_i^{++} contains only the form factor B_i and not the form factor A_i. The first two observations differ from that in Ji et al. and eventually invalidate their argument regarding the consequence of Lorentz invariance, while the last finding is already a well established result [17]. We have also shown that the claim of frame-independence of the results by Ji et al. is invalid. Further, we compared and contrasted two approaches to the spin sum rules, namely, the approach based on light front spin operators advocated by us and the one proposed by Jaffe and Manohar [11] based on matrix elements of the energy–momentum tensor. The physical content of our sum rules is very transparent since the intrinsic quark and gluon contributions appearing in these sum rules are directly related to observables in polarized deep inelastic scattering. In contrast, in the second approach details of the dynamics remain hidden and the separation into orbital and intrinsic spin parts is not visible.

Appendix

All our calculations are performed in light front field theory. We will follow the conventions of Ref. [15]. Let us take the state to be polarized in the +ve x direction. Explicitly, it is given by

$$\big|P, S^{(1)}\big\rangle = \frac{1}{\sqrt{2}}\big(|P, \mathrm{up}\rangle + |P, \mathrm{down}\rangle\big) \tag{30}$$

where $|P, \mathrm{up}\rangle$ and $|P, \mathrm{down}\rangle$ are helicity eigenstates. Then we need to evaluate the matrix elements for up down, down up, up up and down down helicity states.

To simplify the calculations further, we use

$$\overline{U}\big(P, S^{(1)}\big)\sigma^{\mu\nu}U\big(P, S^{(1)}\big) = 2\epsilon^{\mu\nu\alpha\beta}\frac{P_\beta S_\alpha}{M_N}. \tag{31}$$

Note that in our convention, $\epsilon^{+-12} = -2$ and hence Eq. (31) differs from Eq. (5.36) of Ref. [16] by a factor of 2. The components of the polarization vector S^μ are explicitly $S^+ = 0$, $S^1 = M_N$, $S^2 = 0$ and $S^- = 2M_N\frac{P^1}{P^+}$. We present explicit calculations in case II in which we need to calculate the five matrix elements, namely, $\overline{U}(P', S')U(P, S)$ and $\overline{U}(P', S')\gamma^\mu U(P, S)$.

An explicit evaluation of these matrix elements gives the following (in the frame with non-zero $\overline{P}^\perp$):

$\mathbf{S' = up}$, $\mathbf{S = down}$

$$\overline{U}\big(P', S'\big)U(P, S)$$
$$= \frac{1}{\sqrt{P^+P'^+}}\big[\overline{P}^+\big(\Delta^{(1)} - i\Delta^{(2)}\big) - \Delta^+\big(\overline{P}^{(1)} - i\overline{P}^{(2)}\big)\big], \tag{32}$$

$$\overline{U}\big(P', S'\big)\gamma^1 U(P, S) = \frac{M_N}{\sqrt{P^+P'^+}}\Delta^+, \tag{33}$$

$$\overline{U}\big(P', S'\big)\gamma^2 U(P, S) = -i\frac{M_N}{\sqrt{P^+P'^+}}\Delta^+, \tag{34}$$

$$\overline{U}\big(P', S'\big)\gamma^+ U(P, S) = 0, \tag{35}$$

$$\overline{U}\big(P', S'\big)\gamma^- U(P, S) = 2\frac{M_N}{\sqrt{P^+P'^+}}\big(\Delta^{(1)} - i\Delta^{(2)}\big). \tag{36}$$

$\mathbf{S' = down}$, $\mathbf{S = up}$

$$\overline{U}\big(P', S'\big)U(P, S)$$
$$= \frac{1}{\sqrt{P^+P'^+}}\big[-\overline{P}^+\big(\Delta^{(1)} + i\Delta^{(2)}\big) + \Delta^+\big(\overline{P}^{(1)} + i\overline{P}^{(2)}\big)\big], \tag{37}$$

$$\overline{U}(P', S')\gamma^1 U(P, S) = -\frac{M_N}{\sqrt{P^+P'^+}}\Delta^+, \tag{38}$$

$$\overline{U}(P', S')\gamma^2 U(P, S) = -i\frac{M_N}{\sqrt{P^+P'^+}}\Delta^+, \tag{39}$$

$$\overline{U}(P', S')\gamma^+ U(P, S) = 0, \tag{40}$$

$$\overline{U}(P', S')\gamma^- U(P, S) = -2\frac{M_N}{\sqrt{P^+P'^+}}\left(\Delta^{(1)} + i\Delta^{(2)}\right). \tag{41}$$

S′ = up, S = up

$$\overline{U}(P', S')U(P, S) = \frac{1}{\sqrt{P^+P'^+}}\overline{P}^+(2M_N), \tag{42}$$

$$\overline{U}(P', S')\gamma^1 U(P, S) = \frac{1}{\sqrt{P^+P'^+}}\left[\overline{P}^+\left(2\overline{P}^{(1)} - i\Delta^{(2)}\right) - \frac{\Delta^+}{2}\left(\Delta^{(1)} - 2i\overline{P}^{(2)}\right)\right], \tag{43}$$

$$\overline{U}(P', S')\gamma^2 U(P, S) = \frac{1}{\sqrt{P^+P'^+}}\left[\overline{P}^+\left(i\Delta^{(1)} + 2\overline{P}^{(2)}\right) - \frac{\Delta^+}{2}\left(2i\overline{P}^{(1)} + \Delta^{(2)}\right)\right], \tag{44}$$

$$\overline{U}(P', S')\gamma^+ U(P, S) = 2\sqrt{P^+P'^+}, \tag{45}$$

$$\overline{U}(P', S')\gamma^- U(P, S) = \frac{2}{\sqrt{P^+P'^+}}\left[M_N^2 + \left(\overline{P}^\perp\right)^2 - \frac{1}{4}\left(\Delta^\perp\right)^2 + i\left(\overline{P}^{(2)}\Delta^{(1)} - \overline{P}^{(1)}\Delta^{(2)}\right)\right]. \tag{46}$$

S′ = down, S = down

$$\overline{U}(P', S')U(P, S) = \frac{1}{\sqrt{P^+P'^+}}\overline{P}^+(2M_N), \tag{47}$$

$$\overline{U}(P', S')\gamma^1 U(P, S) = \frac{1}{\sqrt{P^+P'^+}}\left[\overline{P}^+\left(2\overline{P}^{(1)} + i\Delta^{(2)}\right) - \frac{\Delta^+}{2}\left(\Delta^{(1)} + 2i\overline{P}^{(2)}\right)\right], \tag{48}$$

$$\overline{U}(P', S')\gamma^2 U(P, S) = \frac{1}{\sqrt{P^+P'^+}}\left[\overline{P}^+\left(-i\Delta^{(1)} + 2\overline{P}^{(2)}\right) - \frac{\Delta^+}{2}\left(-2i\overline{P}^{(1)} + \Delta^{(2)}\right)\right], \tag{49}$$

$$\overline{U}(P', S')\gamma^+ U(P, S) = 2\sqrt{P^+P'^+}, \tag{50}$$

$$\overline{U}(P', S')\gamma^- U(P, S) = \frac{2}{\sqrt{P^+P'^+}}\left[M_N^2 + \left(\overline{P}^\perp\right)^2 - \frac{1}{4}\left(\Delta^\perp\right)^2 + i\left(\overline{P}^{(1)}\Delta^{(2)} - \overline{P}^{(2)}\Delta^{(1)}\right)\right]. \tag{51}$$

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